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Proportions of Renaissance Tenor Flutes and the Relationship of Verona Flutes to Foot-Length Standards

In a previous issue of this Journal, I presented a survey of original consort flutes and a study of their pitches in relation to treatises and musical compositions in which flutes were included.¹ The present article deals with acoustics and mathematics more than with musical issues.

Renaissance flutes have a very simple, cylindrical construction, which allows a simple calculation of the proportion between pitch and sounding length. The function of sounding length in determining the pitch of Renaissance flutes was first stated by Filadelfio Puglisi: 'An approximate idea of pitch can be inferred from their speaking lengths, and it is therefore useful to compare these'² and 'For Renaissance flutes I very much prefer to go by speaking length'.³ Peter Spohr has expressed this relationship in the form of a graph with the linear function $a' = 803 - 0.683L$ where a' is the pitch in Hz and L is the sounding length in millimeters.⁴ While he gave no theoretical justification for this formula, simply graphing his observations of a few copies of

Renaissance flutes,⁵ his results are nevertheless very close to mine, which are based on the non-linear equation used by physicists to calculate the pitch of open cylindrical pipes (see Graph 1).

Table 1 provides data that allow us to calculate pitch in relationship to sounding length for any Renaissance tenor flute.⁶ The simplified version of the physicist's formula is $F = CV/2L$ where F is frequency of the tube in Hz; L is the sounding length; C is a correction factor taking into account flute parameters (mouthhole size, cork position, bore diameter, wall thickness, etc.) and external parameters (player, temperature and humidity), and V is the speed of sound in air (variable with temperature, but about 343 m/s). In theory the formula should take into account the diameter d of the bore, but as I have shown elsewhere⁷ the ratio of sounding length to diameter of bore is fairly constant for Renaissance flutes and is close to 33. It can therefore be neglected in this study of proportions.⁸ Thus if the flute is a tenor in d , the pitch of its a' will be $3/2$ (the ratio

¹ See Philippe Allain-Dupré, 'Renaissance and Early Baroque Flutes: An Update on Surviving Instruments, Pitches and Consort Grouping', *Galpin Society Journal* LVII, (2004), pp.53-61.

² Filadelfio Puglisi, 'A Survey of Renaissance Flutes', *Galpin Society Journal* XLI, (1988), 68.

³ Bruce Haynes, *A History of performing pitch: The story of the A*, (Lanham, Md: Scarecrow Press, 2002), 45.

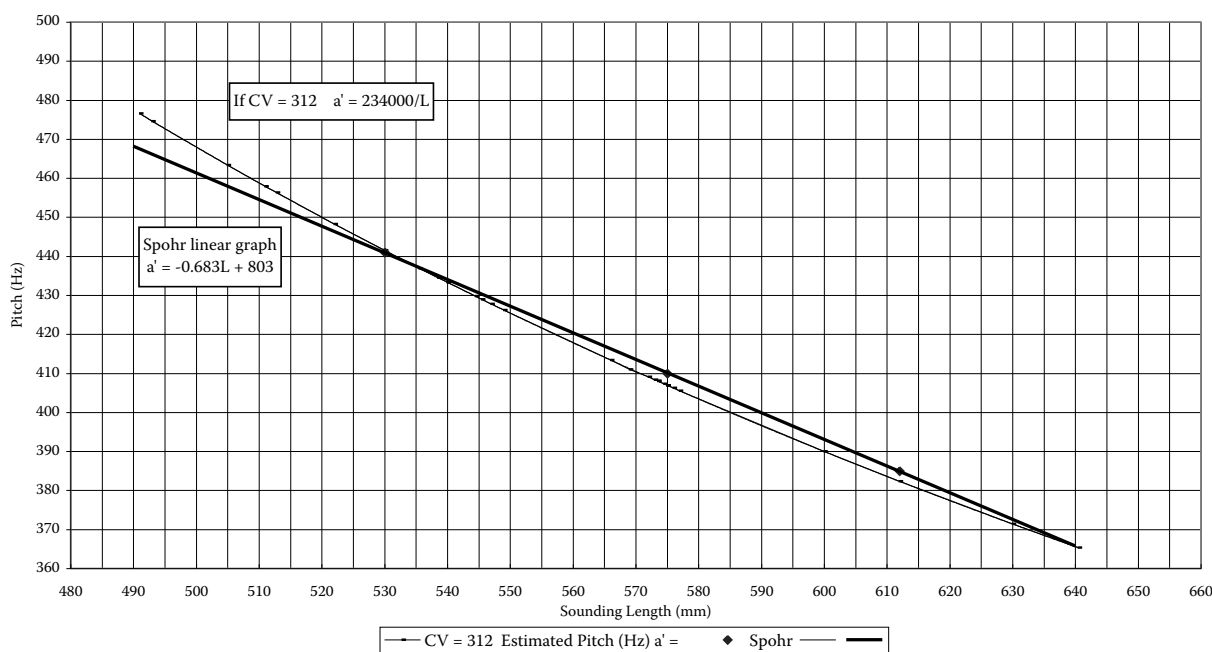
⁴ Peter Spohr, *Transverse flutes down the centuries from all over the world*, (Frankfurt: 1991), 85.

⁵ Personal communication, 2004.

⁶ The author also offers an Excel calculator version at allain-dupre@club-internet.fr

⁷ A graph is available at <http://perso.club-internet.fr/allaindu/fluterenaissance/6-proportions.htm>

⁸ Let us call D the distance from the middle of the embouchure to the cork. Mersenne (*Harmonie Universelle*, Paris 1636, 241) put the cork at 8 lignes=18mm, same distance as the bore of the cylindrical flute and Theobald Boehm, *La flûte et son jeu*, (French translation by T. Labat, Paris, Zurfluh), 12 put it at 17 mm, the same as the bore of the head-joint of his flute. D is equal to the tube diameter d for cylindrical flutes. As demonstrated by Michèle Castellengo in *La flûte à une clef*, bulletin du GAM (Paris 1978), this portion of the tube does not affect the frequency of the fundamental. But an end correction of $0.8 \times$ diameter should be taken in account. This new constant for the speed of sound in air $KV = 2/3 a' \times 2(L+0.8d)$ is proportional to CV , since d is proportional to L in Renaissance flutes, L being approximately $33d$.



Graph 1. Proportions of cylindrical tenor flutes.

of the fifth) $\times F$. The relationship between pitch and length is a constant $CV = 2/3 a' \times 2L$. To calculate CV I have chosen a few remarkable flutes: the high-pitched SAM 174 (estimated by Boaz Berney at 480 Hz), the B-1065 Bassano at 413 Hz,⁹ the trefoil flutes in Verona at 407 Hz,¹⁰ the B-1066 Rafi at 382 Hz¹¹ and the Bologna Rafi that I measured at 406 Hz. They give a value for CV of around 312, 313 or 314 m/s, a little less than $V = 343$ m/s. Thus the formula for calculating the frequency of a' , based upon total sounding length of d' , is $a' = 3/2 F = 3/4 \times CV/L$. In Table 1 the actual measurements are printed normally, while the results of calculation, using three different estimates for CV are printed in **bold italic**. The first column, with a constant of 312 m/s, reflects many of my observations made in museums when the flutes were cold. The two other columns give more realistic results, when the flute is warm and played in ensemble. These pitches follow Puglisi's estimates, which are 410 Hz and 435 Hz for the two largest groups.¹² This method of pitch calculation gives results very close to the observations I have made in Italian collections and in the Musée Instrumental de Bruxelles. It helps in determining

the pitches of flutes in collections such as those in Austria, where current policy prevents the playing of even a single note on original instruments to estimate their pitch. It is also helpful for estimating the pitch of damaged flutes (such as B-Brussels: 1065 and 1064; NL-Amsterdam: Rijksmuseum NG NM 7692) or unplayable flutes (such as I-Rome: Museo dSM 0712, where the cork is glued in the wrong position).

This method shows itself reliable for calculating the pitch of Renaissance flutes, in which all fingerings for D and for A octave are simple harmonics, or partials, of the tube. [Acousticians, such as Benade, distinguish between partials of the tone being produced—in which the components are always harmonic—and vibrational modes of the bore; the latter are rarely exactly harmonic, but they are considered to be so in Renaissance flute fingering charts¹³] In my experience with baroque, simple system and Boehm flutes, on the other hand, the problem may arise that the distance between the lowest tone hole and the end of the flute may show considerable variation among flutes playing at the same (average) pitch. For various reasons the bottom

⁹ Estimate by David Lasocki, *Traverso* Vol.16, (January 2004), 4.

¹⁰ Measurements by the author in the *Accademia Filarmonica di Verona* 1988, 1990 and 1992.

¹¹ Philippe Allain-Dupré, *Les flûtes de RAFI*, (Courlay: J. M. Fuzeau, 2000), 20.

¹² Filadelfio Puglisi, 'A Survey of Renaissance Flutes', *Galpin Society Journal* XLI, (1988), 71.

¹³ See Agricola, *Musica Instrumentalis Deudsch*, (1545), 172-175 (Leipzig: pseudo-facsimile by Breitkopf & Härtel, 1896), and Virgiliano, *Il Dolcimelo*, (Florence: facsimile by SPES, 1979), folio 54. A modern summary of these fingering charts is available at <http://allaindu.club.fr/fluterenaissance/Tablature.PDF>

	Flute and location <i>(hypothetical flutes which are not historical data are shown in Italics)</i>	Stamp	Sounding length (mm) SL=	Total length (mm) TL=	Ratio TL/SL	CV=312	CV=313	CV=314
						Estimated Pitch (Hz) a'=	Estimated Pitch (Hz) a'=	Estimated Pitch (Hz) a'=
	A-Vienna: SAM 174 (was A185)	!!!	491	577	1.18	477	478	480(BB)
	R-St Petersburg: 438 ex Snoeck		493	578	1.17	475	476	478
Cornet- Thon or Mezzo punto	A-Vienna: SAM 176 (was C187)	LISSIEV	505	601	1.19	463	465	466
	D-Berlin: 2663 ex Snoeck		511	609	1.19	458	459	461
	R-St Petersburg 437 ex Snoeck		513	600	1.17	456	458	459
	<i>Mersenne, Harmonie Universelle (1636)</i>		522	605	1.16	448	450	451
440 Hz	<i>Modern prototype</i>		530			442(PAD)	443	444
	NL-Amsterdam: Rijksmuseum NG NM 7692	^/+	535.5	638.5	1.19	437	438	440
Chor Thon + 1/2 ton or tutto punto	I-Verona: Biblioteca Capitolare 5	AA	538.5	630	1.17	435	436	437
	I-Verona: Biblioteca Capitolare 6	AA	540	628	1.16	433	435	436
	I-Verona: Biblioteca Capitolare 1	crowned eagle	540	629	1.16	433	435	436
	I-Verona: Biblioteca Capitolare 3	!!!	544.5	631	1.16	430	431	433
	I-Verona: Biblioteca Capitolare 2	!!!	545.5	632.5	1.16	429	430	432
	CH-Basel: HM 1907.1980	!!!	547	645	1.18	428	429	431
	I-Verona: Biblioteca Capitolare 4	C.RAFI	549	648	1.18	426	428	429
Tenor/alt in d' at Chor Thon or Tuono Corista (one tone below Cornet Thon)	D-Berlin: 5422		566	650	1.15	413	415	416
	B-Brussels: 1065	!!!	569	664	1.17	411	413(DL)	414
	B-Brussels: 1064	!!!	572	664	1.17	409	410	412
	I-Rome: Museo dSM 0715 (was 2791)	A or AA	573	660.5	1.15	408	410	411
	I-Rome: Museo dSM 0714 (was 2790)		573.5	670.5	1.17	408	409	411
	A-Vienna: SAM 175 was C[atajo] 186		573.5	671.5	1.17	408	409	411
	I-Verona: Acc. Filarmonica 13282	trefoil	574.5	658.5	1.15	407	409	410
	I-Verona: Acc. Filarmonica 13283	trefoil	575	684	1.19	407	408	410
	I-Verona: Acc. Filarmonica 13284	trefoil	575	683	1.19	407(PAD)	408	410
	I-Verona: Acc. Filarmonica 13285	trefoil	575	684	1.19	407(PAD)	408	410
	I-Verona: Acc. Filarmonica 13286	trefoil	575	684	1.19	407	408	410
	I-Bologna: Museo Civico 3288	C.RAFI	576	682.5	1.18	406(PAD)	408	409
	I-Rome: Museo dSM 0712 (was 2789)	C.RAFI	577	688	1.19	406	407	408
Deeper pitches	<i>Modern prototype in d at 392</i>		600			390	391	393
	B-Brussels 1066	C.RAFI	612	717	1.17	382(PAD)	384	385
	<i>Praetorius, Syntagma Musicum pl.IX</i>		630			371	373	374
	I-Verona: Acc. Filarmonica 13287	[C?] RAFI	640.5	734.5	1.15	365	367	368

Table 1. List of Renaissance tenor flutes with estimates of their pitch taking their sounding length into account.

D on baroque flutes is sometimes very flat to the rest of the instrument, and the bottom C on Boehm flutes is often sharp. Thus, calculating the pitch of later flutes from the length of the complete tube would not be a reliable approach.

To consider the effect on pitch of four centuries of

shrinkage, let us assume that the original dimensions were 1mm greater in length and 1 mm wider in bore (these values are in fact overestimates for purposes of argument, the actual amount of shrinkage probably being considerably smaller). As demonstrated by Mahillon¹⁴ an increase of 1 mm in bore is equivalent

¹⁴ Victor Charles Mahillon, *Éléments d'acoustique musicale et instrumentale*, (Brussels: Les amis de la musique, 1984), 90.

SL	TL	ratio	estimated pitches (Hz)		
510.3	597	1.17	459	460	462
505.9	597	1.18	463	464	465

Table 2. *Calculated Pitches of Flutes in a Futteral at Augsburg.*

this Futteral represents a unique testimony that consort flutes were once built to match the common pitch of other woodwinds. Note that the Lissieu flute, a late seventeenth century French flute, and D-Berlin 2663, roughly made and probably military, cannot be considered examples of consort flutes. Information is not available for R-St Petersburg 437 ex-Snoeck.

The Relationship of Verona Flutes to Foot-Length Standards: a Rule of Proportions

Table 1 raises the question of why there are so many flutes at around 406-414 Hz. When I was a newcomer to Renaissance woodwinds, my first aim was to build exact replicas of original instruments I had seen in Museums. In 1990 I was very proud to show my replica of a flute consort based upon the trefoil flutes in the Accademia Filarmonica, Verona, with its unexpected pitch of 408 Hz. My colleague Jean-Noël Catrice brought a recorder consort at original pitch made by Bob Marvin after recorders in Vienna. When we put our instruments side by side, there was a revelation for me: my tenor flutes in d' had the same sounding length as his tenor recorders in c', and there was the same relationship for bass flutes in G and basset recorders in F. Those lengths were respectively two and three German feet (The Brunswick foot at 285 mm, which corresponds to the scale of Praetorius's plates in *Syntagma Musicum* II; the Frankfurt and Wurtemberg foot, both at 286.5 mm; and the Bavarian foot at 288 mm). Then I discovered the same with cornetts [or cornettos]: modern replicas made at a=440 are 600 mm long, which is also the length of a modern flute replica in c' at 440 (or, considered more historically, in d' at 392). Most original cornetts are between 560 and 580 mm long, i.e. 2 feet,¹⁷ which makes them pitched at Cornet-Thon (these being measurements of the instrument itself, without the mouthpiece). Thus we can express the practice of sixteenth-century

woodwind makers when making instruments at the usual Cornet-Thon pitch as a set of rules:

- Renaissance descant recorders in c'' are 1 foot in sounding length, tenor recorders in c' are 2 feet, bassets in f are 3 feet and basses in c are 4 feet.
- Cornettos in a are 2 feet in sounding length.
- Renaissance tenor flutes are 2 feet; basses, 3 feet. Being in d' and g (rather than c' and f), they therefore sound one tone lower than Cornet-Thon. This pitch correspond to Praetorius's Chor-Thon, or Italian corista, which was a tone lower than the high Cammerton or Mezzo Punto.

We can also find instances in which a concern for length standards also applies to modern instruments: In Japan the Shakuhachi flute bears the name of its length, shaku (one foot) hachi (eight), which is 545 mm (for a Shakuhachi in d' at 440 Hz); In France there is still a tradition naming bagpipes chanters by their size in pouces.¹⁸

Now let us consider Table 3 in which the sounding lengths of Northern Italian flutes have been compared to multiples of foot measurements. The trefoil flutes follow the rules defined above. The two Rafi flutes in the Academia Filarmonica are pitched one tone lower, so I have chosen the ratio of the major tone (9/8) as the multiplier for the foot-measurement as the basis of comparison for these instruments. 13278 AF and all the BC Flutes are at tutto punto, a semitone above Corista, or more accurately a semitone below Mezzo Punto. Here I have chosen as a divisor the ratio of the semitone of fretted instruments, in which the semitones are equal.¹⁹ Mersenne gives this ratio for the viola da gamba at 1.05945, almost the same as the 12th root of 2 of equal temperament (1.05946).²⁰ Then I have chosen three values for the German foot:

1. The Brunswick foot at 285 mm, which corresponds to the scale given on Praetorius's plates in *Syntagma Musicum* II
2. The Frankfurt and Wurtemberg foot, both at 286.5 mm

¹⁷ Edward H. Tarr, *Ein katalog erhaltener Zinken*, in 'Zink un Posaune. Studien zu Uberlieferung, Instrumentbau und Repertoire', *Basler Jahrbuch für Historische Musikpraxis* V, (1981), 11-262.

¹⁸ Bernard Blanc, 'Les cornemuses du centre, filiation et évolution 28-31', in *Les cornemuses de George Sand. Autour de Jean Sautivet, fabricant et joueur de musette dans le Berry (1796-1867)*, (Montluçon, Musées de Montluçon, 1996).

¹⁹ Artusi, *overo delle imperfettioni della moderna musica*, (Venice, 1600, facsimile by FORNI, Bologna), 11.

²⁰ Marin Mersenne, *Harmonie Universelle*, (Paris, 1636, facsimile CNRS, 1969), Livre quatriesme, Proposition VII.

3. The Bavarian foot at 288 mm called also Alt-Kulmische Maß, available since 1233.

Table 3 gives in italics the proportions which show more than 1% deviation from theoretical lengths based upon foot measurements. In bold are the more plausible proportions, closest to 100%! Of course one might contest the choice of these particular values, which are close to each other, instead of the Nuremberg foot (304 mm), the Roman once (298 mm) or the Venice foot itself (347 mm). Anthony Baines has pointed out that the success of Venetian instruments in German courts led to some degree of standardisation in instrumental playing pitch.²¹ This standardisation led to the adoption of the most common values of foot measurement for German customers. Close to the values I have chosen here are also the Amsterdam foot and the Saxon foot, both at 283.2 mm. Incidentally, the ratio between the Bavarian foot and the French pied du Roi is $288/324 = 8/9$, the ratio of the major whole tone. Thus the two Ravi flutes made in 1546 in Lyon are by chance exactly at one tone lower than the German trefoil flutes. I can explain the sounding length of the Rome and Bologna Ravi flutes as representing special orders from foreign musicians who ordered flutes in Lyon at the same sounding length as their cornetts from Venice (most of these are 576 mm long). I have no explanation for the slightly longer sounding length of the Ravi flute in the Biblioteca Capitolare, nor for the shorter AA bass, which may have been shortened a little.²² But concerning all the other flutes, the trefoil tenor flutes, probably made in Munich or in the Verona area, match the Bavarian foot, and the Crowned Eagle flutes match the Frankfurt foot. The !!! flutes, probably made in Venice, also match the Bavarian foot, and the AA flutes match the Württemberg or Frankfurt foot.

The rules stated above allows us to calculate the sounding length of incomplete flutes, such as A-Vienna: SAM 207 (formerly C[atajo] 218). Only the body of this flute survives, and it was misidentified as a bassanello by Schlosser in his

1920 catalogue.²³ It has been estimated as an A-flute at tutto punto; its sounding length should therefore be Foot $\times (8/3)/1.05946$ because $8/3$ is the ratio of a 2-foot flute a fourth ($4/3$) below, and $1/1.05946$ is the factor to raise the pitch by a semitone. If we take the foot to be 285 mm, the result is 717.3 mm, very close to the estimates that flutemakers arrive at by empirical methods. Friedrich Von Huene, for instance, made a copy at 712.5 mm, and I made one at 720 mm.²⁴ More recently I have made a reconstruction of this flute with a sounding length of 717.3 mm which has proved satisfactory.²⁵

I recognise that this attempt to explain the sounding lengths of Renaissance flutes and their unexpected pitches compared to other Renaissance woodwinds is somewhat speculative. In the Renaissance, frequency of vibration was not used to define pitch as it is now, but if one considers these physical relationships not merely as coincidences, it is possible to explain how wood-turners and organ-makers established a standard pitch for their instruments. Further research is still needed, specially concerning recorders, curved and mute cornets, and organ pipes of seventeenth century, if some still survive in their original condition. This research may change our perception of sixteenth and early seventeenth century pitches, which is often described as chaotic, because the values to be compared are provided by different players in different conditions. Accurate data concerning the sounding length (in feet) may reveal that there was much more uniformity and standardisation than is commonly believed.

I am very much indebted to Filadelfio Puglisi and to Peter Spohr, not only for their former publications (see footnotes 2 and 4) which served as basis for this study, but also for their encouragements during the redaction of this article. I am also very much indebted to Rick Wilson who helped me with the mathematics of Table 1, and to Herbert Myers for his corrections and judicious discussion of this article.

²¹ Anthony Baines, *Woodwind Instruments and their History*, (London: Faber, 1957), 241-42.

²² According to Boaz Berney, personal communication, October 2005.

²³ Julius Schlosser, *Sammlung alter Instrumente*, (Vienna, 1920).

²⁴ See <http://allaindu.club.fr/fluterenaissance/7-bassanello.htm>

²⁵ An audible example of this reconstruction is available at <http://allaindu.club.fr/fluterenaissance/Cima.mp3>

Location	Inv No.	Stamp (M = Maggie Kilbey)	Wood	Size	Pitch Corista	Sounding length	Relationship to foot	Multiple of foot measurement (if foot is 285 mm)	as a % if a foot is 285 mm	as a % if a foot is 286.5 mm	as a % if a foot is 288 mm
Milan		trefoil	box	bass	g	855.0	3	855	100.0	99.5	99.0
	13282	trefoil	box	tenor	d'	574.5	2	570	100.8	100.3	99.7
	13283	trefoil	box	tenor	d'	575.0	2	570	100.9	100.3	99.8
	13284	trefoil	box	tenor	d'	575.0	2	570	100.9	100.3	99.8
	13285	trefoil	box	tenor	d'	575.0	2	570	100.9	100.3	99.8
	13286	trefoil	box	tenor	d'	575.0	2	570	100.9	100.3	99.8
	13287	[C1?],RAFI/lion	plum	tenor	d' - 1	640.5	2 x 9/8	641	99.9	99.4	98.8
	13278		plum	bass	g + 1/2 tutto punto	808.0	3/1.05946	807	100.1	99.6	99.1
	13279	trefoil	box	bass	g	855.0	3	855	100.0	99.5	99.0
	13277	trefoil	box	bass	g	855.5	3	855	100.1	99.5	99.0
	13276	trefoil	box	bass	g	856.0	3	855	100.1	99.6	99.1
13281	Lion	plum	bass	g - 1	964.5	3 x 9/8	962	100.3	99.7	99.2	
				tutto punto		factor / 1.05946					
Biblioteca Capitolare	5	Rafi	box	tenor	d' + 1/2	549.0	2/1.05946	538	102.0	101.5	101.0
	6	AA	maple	tenor	d' + 1/2	539.0	2/1.05946	538	100.2	99.7	99.1
	7	AA	maple	tenor	d' + 1/2	540.0	2/1.05946	538	100.4	99.8	99.3
	8	Eagle	box	tenor	d' + 1/2	540.0	2/1.05946	538	100.4	99.8	99.3
	9	!! M type B	box	tenor	d' + 1/2	544.5	2/1.05946	538	101.2	100.7	100.2
	10	!! M type B	box	tenor	d' + 1/2	544.5	2/1.05946	538	101.4	100.9	100.3
	3	!! M type B	box	bass	g + 1/2	816.0	3/1.05946	807	101.1	100.6	100.1
	4	AA	maple	bass	g + 1/2	795.5	3/1.05946	807	98.6	98.1	97.5

Table 3. Milan and Verona Flutes and their relationships to Foot-Length Standards.

